Technical Tip

The "Solver" Spreadsheet Feature Applied to the Load Assignment Problem

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The information in this document is a supplement to material included in the book:

Truckload Transportation: Economics, Pricing & Analysis
written by Leo J. Lazarus

The book and actual spreadsheet model are available at:
www.TruckloadTransportation.com

! *Technical Tip* – Linear Programming (Solver) Spreadsheet Feature

The solution to the truck and load assignment optimization problem presented in Chapter 6 of *Truckload Transportation: Economics, Pricing and Analysis* was developed using the "Solver" spreadsheet feature. This expanded technical tip details the exact design and setup of the optimization solution in the spreadsheet. The complete context of this problem is not shown here but is included in Chapter 6 of the book.

Initial Setup

The first step is to design the basic formulas that represent the relationship in the objective equation. In this case, the objective is to minimize the total miles needed to assign each available truck to each available load. Table 1 beginning in row 2 lists the amount of empty miles associated with each tractor and load combination. Table 2 represents the values for the 25 decision variables in cells E15 to I19.

	B	C	D	E	F	G	H	I	J
2			**Current Network**	Empty Miles from Tractor Location to Load Pick-up Location					
3				**Load Number**					
4			**Tractor Number**	A	B	C	D	E	
5			1	95	150	60	77	123	
6			2	210	160	140	110	190	
7			3	41	15	79	101	40	
8			4	88	91	115	62	55	
9			5	350	210	140	120	105	
10									
11									
12			**Optimized Solution**	Load Assignments					Total
13				**Load Number**					Truck
14			**Tractor Number**	A	B	C	D	E	Assignments
15			1	1	0	0	0	0	1
16			2	0	1	0	0	0	1
17			3	0	0	1	0	0	1
18			4	0	0	0	1	0	1
19			5	0	0	0	0	1	1
20			Load Assignments	1	1	1	1	1	5
21									
22									
23			**Solution Results**	Total Empty Miles					Total
24				**Load Number**					Empty
25			**Tractor Number**	A	B	C	D	E	Miles
26			1	95	-	-	-	-	95
27			2	-	160	-	-	-	160
28			3	-	-	79	-	-	79
29			4	-	-	-	62	-	62
30			5	-	-	-	-	105	105
31			Miles	95	160	79	62	105	**501**

The values in cells E20 to I20 represent the maximum value of "1" in the load availability constraints. Likewise, the values in cells J15 to J19 represent the maximum value of "1" in the truck availability constraints.

Table 3 beginning in row 23 simply computes the total number of empty miles represented in the solution as shown in Table 2. Beginning in cell E26, the empty mile values in Table 1 are multiplied by the corresponding solution variable value in Table 2. The table below illustrates all the formulas for the design.

	D	E	F	G	H	I	J
2	**Current Network**	Empty Miles from Tractor Location to Load Pick-up Location					
3		**Load Number**					
4	**Tractor Number**	A	B	C	D	E	
5	1	95	150	60	77	123	
6	2	210	160	140	110	190	
7	3	41	15	79	101	40	
8	4	88	91	115	62	55	
9	5	350	210	140	120	105	
10							
11							
12	**Optimized Solution**	Load Assignments					Total
13		**Load Number**					Truck
14	**Tractor Number**	A	B	C	D	E	Assignments
15	1	0	0	0	1	0	=SUM(E15:I15)
16	2	0	1	0	0	0	=SUM(E16:I16)
17	3	1	0	0	0	0	=SUM(E17:I17)
18	4	0	0	0	0	1	=SUM(E18:I18)
19	5	0	0	1	0	0	=SUM(E19:I19)
20	Load Assignments	=SUM(E15:E19)		=SUM(G15:G19)			=SUM(J15:J19)
21							
22							
23	**Solution Results**	Total Empty Miles					Total
24		**Load Number**					Empty
25	**Tractor Number**	A	B	C	D	E	Miles
26	1	=E15*E5	=F15*F5	=G15*G5	=H15*H5	=I15*I5	=SUM(E26:I26)
27	2	=E16*E6	=F16*F6	=G16*G6	=H16*H6	=I16*I6	=SUM(E27:I27)
28	3	=E17*E7	=F17*F7	=G17*G7	=H17*H7	=I17*I7	=SUM(E28:I28)
29	4	=E18*E8	=F18*F8	=G18*G8	=H18*H8	=I18*I8	=SUM(E29:I29)
30	5	=E19*E9	=F19*F9	=G19*G9	=H19*H9	=I19*I9	=SUM(E30:I30)
31	Miles	=SUM(E26:E30)		=SUM(G26:G30)			**=SUM(J26:J30)**

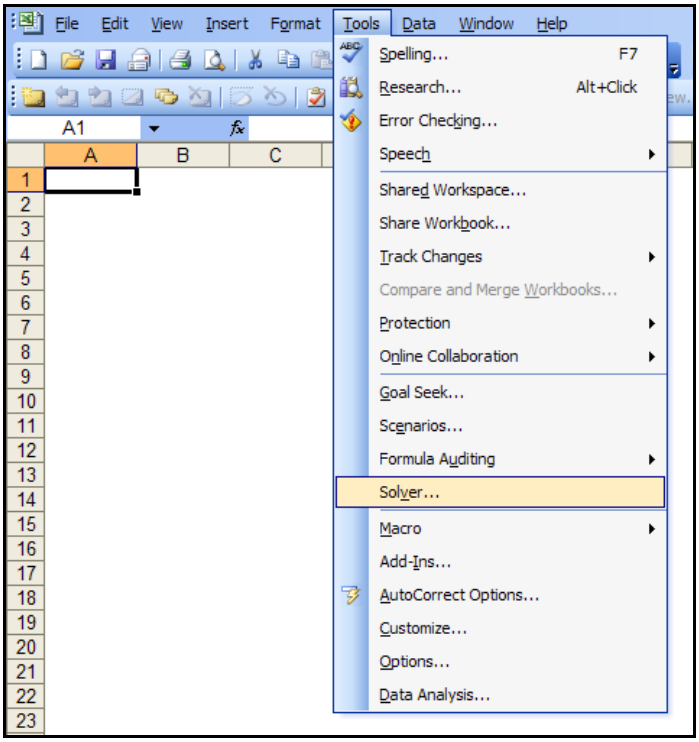
For space considerations, the formulas in rows 20 and 31 are not shown for columns F, H and I. The actual formulas are relatively the same as those shown for columns E and G.

The total empty miles shown in cell J31 of Table 3 represent the final value for the objective equation, the total number of empty miles represented in the proposed solution as shown in Table 2. Cell J31 will recalculate as the suggested solution is changed in cells E15 to I19 of Table 2.

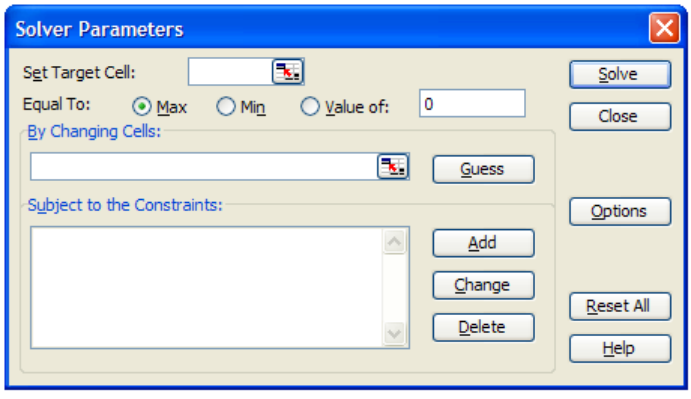
Solver Design

Once the formulas are in place, the next step is to define the objective equation and constraints within the Solver functionality. The picture below shows where to the find the Solver functionality in the spreadsheet.[1]

Accessing the Solver Function in Excel®



After activating the Solver, the Solver Parameters box below will appear.

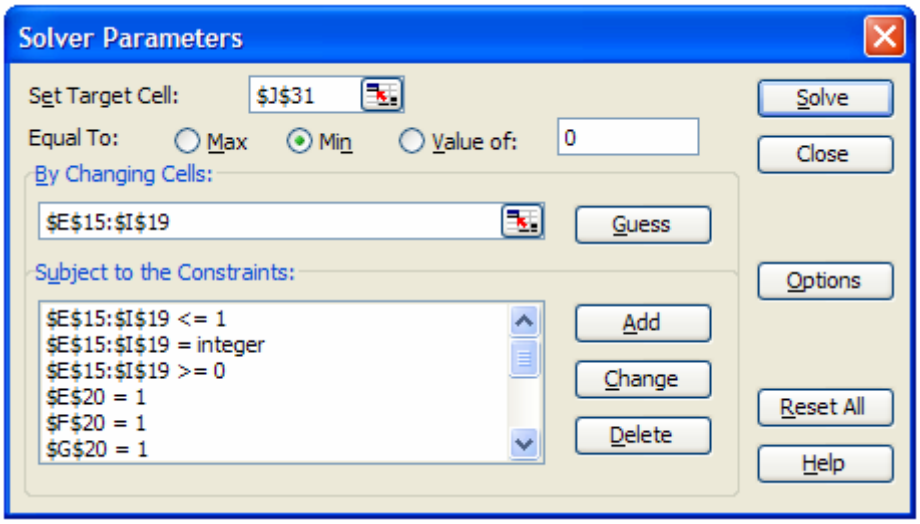


This box will store all the information for the objective equation and all constraints.[2]

[1] In some installations of Excel®, the Solver may not be immediately available. The Solver may need to be set up using "Add-Ins" or a custom installation. Consult the Excel® help feature for more information. The example as shown is based on the tools in Microsoft Excel® 2003.

[2] All Microsoft® product screen shot reprinted with permission from Microsoft® Corporation.

The Solver Parameters box for the load assignment problem is shown below. The box contains places for the objective equation, the decision variables, and all constraints. The objective equation is referenced in the "Set Target Cell" section. The decision variables are referenced in the "By Changing Cells" section. The constraints are represented in the "Subject to the Constraints" section. The box as shown can only display the first six constraints.



Not all constrains are shown in the Solver Parameters box above. All constraints will be shown in the sections that follow. The relationship of the Solver Parameters box to the spreadsheet design model is explained in detail in the sections that follow.

Objective Equation

The objective equation, in its original form, is shown below.

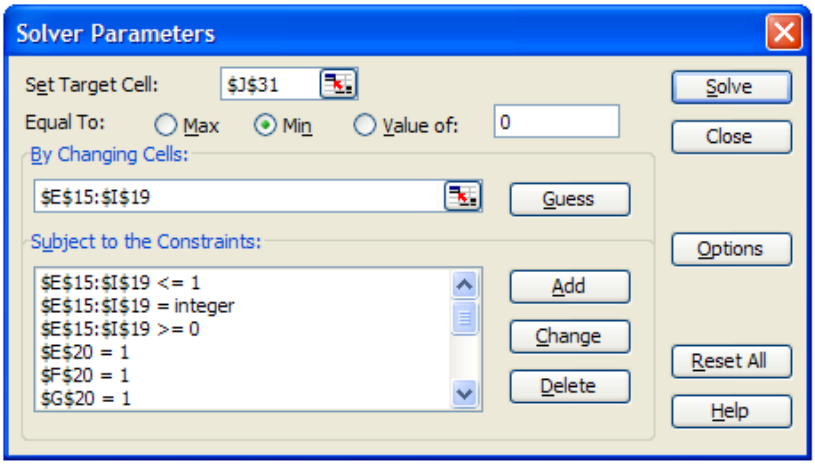
$$\begin{aligned}\text{Minimize: } & 95X_{1A} + 150X_{1B} + 60X_{1C} + 77X_{1D} + 123X_{1E} \\ & + 210X_{2A} + 160X_{2B} + 140X_{2C} + 110X_{2D} + 190X_{2E} \\ & + 41X_{3A} + 15X_{3B} + 79X_{3C} + 101X_{3D} + 40X_{3E} \\ & + 88X_{4A} + 91X_{4B} + 115X_{4C} + 62X_{4D} + 55X_{4E} \\ & + 350X_{5A} + 210X_{5B} + 140X_{5C} + 120X_{5D} + 105X_{5E}\end{aligned}$$

The calculation demonstrated in the original objective equation above is represented in the spreadsheet design shown below in the Solution Results table (Refer to complete spreadsheet design on previous page if needed).

Objective Equation

	B	C	D	E	F	G	H	I	J
21									
22									
23			**Solution Results**	Total Empty Miles					Total
24				**Load Number**					Empty
25			**Tractor Number**	A	B	C	D	E	Miles
26			1	=E15*E5	=F15*F5	=G15*G5	=H15*H5	=I15*I5	=SUM(E26:I26)
27			2	=E16*E6	=F16*F6	=G16*G6	=H16*H6	=I16*I6	=SUM(E27:I27)
28			3	=E17*E7	=F17*F7	=G17*G7	=H17*H7	=I17*I7	=SUM(E28:I28)
29			4	=E18*E8	=F18*F8	=G18*G8	=H18*H8	=I18*I8	=SUM(E29:I29)
30			5	=E19*E9	=F19*F9	=G19*G9	=H19*H9	=I19*I9	=SUM(E30:I30)
31			Miles	=SUM(E26:E30)		=SUM(G26:G30)			**=SUM(J26:J30)**

The first formula in cell E26 (=E15*E5) directly corresponds to the first element in the objective equation ($95X_{1A}$). X_{1A} corresponds to cell E15 and the 95 corresponds to cell E5 that displays the 95 empty miles between Tractor 1 and Load A. The grand total sum in cell J31 represents the value represented by the entire objective equation.



In the Solver Parameters box in the spreadsheet, the "Set Target Cell" box points to cell J31 to identify the objective equation result for the Solver. The other cells of the Solver Parameters box are explained in the sections that follow.

Decision Variables

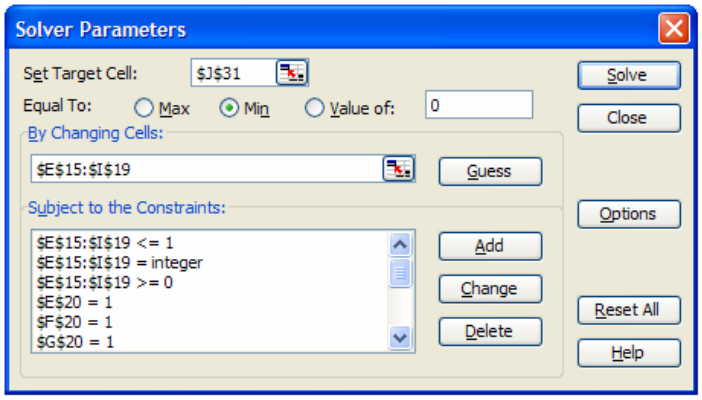
As a reference, the original decision variables for Truck 1 and Truck 2 are shown below.

X_{1A} = Truck 1 assigned to Load A X_{2A} = Truck 2 assigned to Load A
X_{1B} = Truck 1 assigned to Load B X_{2B} = Truck 2 assigned to Load B
X_{1C} = Truck 1 assigned to Load C X_{2C} = Truck 2 assigned to Load C
X_{1D} = Truck 1 assigned to Load D X_{2D} = Truck 2 assigned to Load D
X_{1E} = Truck 1 assigned to Load E X_{2E} = Truck 2 assigned to Load E

Each decision variable must be represented in the spreadsheet decision model in an individual cell. For this problem, the decision variables are represented in cells E15 through I19 as shown below.

	D	E	F	G	H	I	J
12	**Optimized Solution**	Load Assignments					Total
13		**Load Number**					Truck
14	**Tractor Number**	A	B	C	D	E	Assignments
15	1	0	0	0	1	0	=SUM(E15:I15)
16	2	0	1	0	0	0	=SUM(E16:I16)
17	3	1	0	0	0	0	=SUM(E17:I17)
18	4	0	0	0	0	1	=SUM(E18:I18)
19	5	0	0	1	0	0	=SUM(E19:I19)
20	Load Assignments	=SUM(E15:E19)		=SUM(G15:G19)			=SUM(J15:J19)

Decision variable X_{1A} corresponds to cell E15 in the spreadsheet design. X_{2A} corresponds to cell E16 in the spreadsheet design. X_{5E} corresponds to cell I19 in the spreadsheet design. Each additional decision variable value for each of the five trucks and five loads has a corresponding location in the spreadsheet design. The decisions variables are referenced in the Solver as shown below.



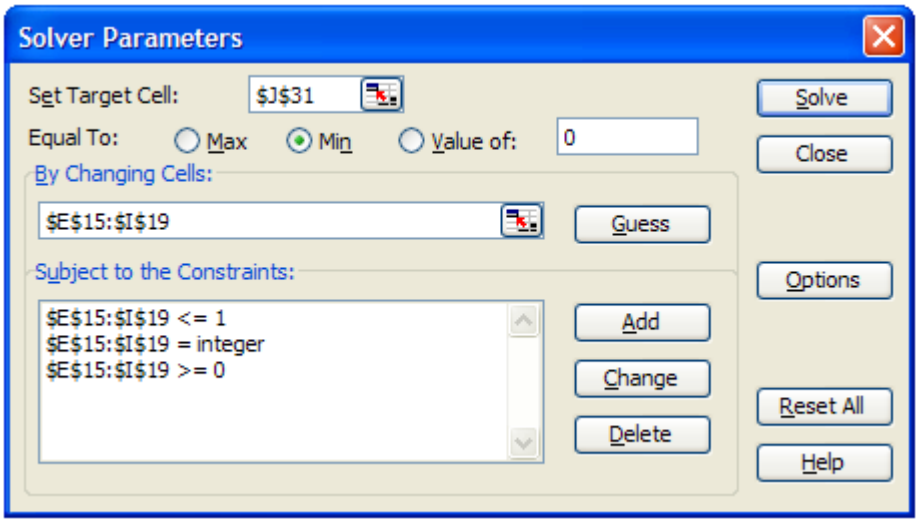
The decision variables range is represented as $E$15:$I$19 in the "By Changing Cells" section of the Solver Parameters box. All decision variables must be in adjoining cells in the spreadsheet design.

Decision Variable Constraints

The decision variable constraints are shown below. The purpose of these constraints is to force each decision variable to assume an exact value of either "0" or "1." A value of "1" indicates that the tractor and load are paired together and a value of "0" indicates that the tractor and load are not paired together.

Non-Negativity: $X_{1A}\ X_{2A}\ X_{3A}\ .\ .\ .\ X_{5E} \geq 0$
Maximum Value of 1: $X_{1A}\ X_{2A}\ X_{3A}\ .\ .\ .\ X_{5E} \leq 1$
Integer Requirement: $X_{1A}\ X_{2A}\ X_{3A}\ .\ .\ .\ X_{5E}$ = Integer

The Solver Parameters box below displays the setup of these three constraints in the spreadsheet design. The first constraint ($E$15:$I$19 <=1) indicates that any value in each individual cell in the range must be less than or equal to 1. The next two constraints are the integer constraint and non-negativity constraint, indicating that the values in these cells must be an integer and must be greater than or equal to 0.



Each constraint references the decision variables located in cells E15 to I19.

	B	C	D	E	F	G	H	I	J
10									
11									
12			**Optimized Solution**	Load Assignments					Total
13				**Load Number**					Truck
14			**Tractor Number**	A	B	C	D	E	Assignments
15			1	0	0	0	1	0	=SUM(E15:I15)
16			2	0	1	0	0	0	=SUM(E16:I16)
17			3	1	0	0	0	0	=SUM(E17:I17)
18			4	0	0	0	0	1	=SUM(E18:I18)
19			5	0	0	1	0	0	=SUM(E19:I19)
20			Load Assignments	=SUM(E15:E19)		=SUM(G15:G19)			=SUM(J15:J19)

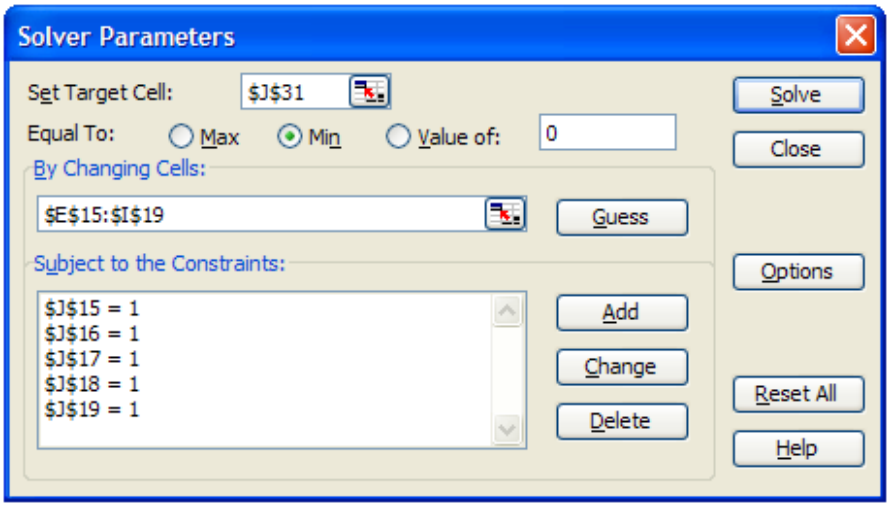
Truck Availability / Capacity Constraints

The truck capacity constraints work with the decision variable constraints so that each available tractor is assigned to only one available load.

Truck Availability / Capacity Constraints:

$$\text{Truck 1 Capacity:} \quad X_{1A} + X_{1B} + X_{1C} + X_{1D} + X_{1E} = 1$$
$$\text{Truck 2 Capacity:} \quad X_{2A} + X_{2B} + X_{2C} + X_{2D} + X_{2E} = 1$$
$$\text{Truck 3 Capacity:} \quad X_{3A} + X_{3B} + X_{3C} + X_{3D} + X_{3E} = 1$$
$$\text{Truck 4 Capacity:} \quad X_{4A} + X_{4B} + X_{4C} + X_{4D} + X_{4E} = 1$$
$$\text{Truck 5 Capacity:} \quad X_{5A} + X_{5B} + X_{5C} + X_{5D} + X_{5E} = 1$$

Each truck capacity constraint is represented in the Solver Parameters box as shown below in the "Subject to the Constraints" section.



Each constraint references the row and cell with the sum of the decision variables for each available tractor in column J. Notice in the solution that each available tractor row has four "0" values and only one "1" value, indicating that each tractor has been assigned to only one available load.

	B	C	D	E	F	G	H	I	J
10									
11									
12			**Optimized Solution**	Load Assignments					Total
13				**Load Number**					Truck
14			**Tractor Number**	A	B	C	D	E	Assignments
15			1	0	0	0	1	0	=SUM(E15:I15)
16			2	0	1	0	0	0	=SUM(E16:I16)
17			3	1	0	0	0	0	=SUM(E17:I17)
18			4	0	0	0	0	1	=SUM(E18:I18)
19			5	0	0	1	0	0	=SUM(E19:I19)
20			Load Assignments	=SUM(E15:E19)		=SUM(G15:G19)			=SUM(J15:J19)

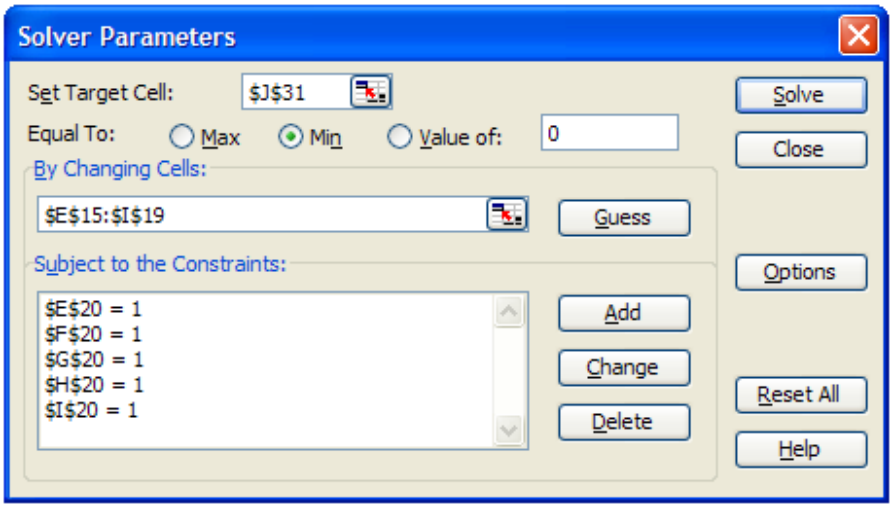
Load Availability / Demand Constraints

The load availability constraints work with the decision variable constraints so that only one tractor is assigned to each available load.

Load Availability / Demand Constraints:

$$\text{Load A Availability:} \quad X_{1A} + X_{2A} + X_{3A} + X_{4A} + X_{5A} = 1$$
$$\text{Load B Availability:} \quad X_{1B} + X_{2B} + X_{3B} + X_{4B} + X_{5B} = 1$$
$$\text{Load C Availability:} \quad X_{1C} + X_{2C} + X_{3C} + X_{4C} + X_{5C} = 1$$
$$\text{Load D Availability:} \quad X_{1D} + X_{2D} + X_{3D} + X_{4D} + X_{5D} = 1$$
$$\text{Load E Availability:} \quad X_{1E} + X_{2E} + X_{3E} + X_{4E} + X_{5E} = 1$$

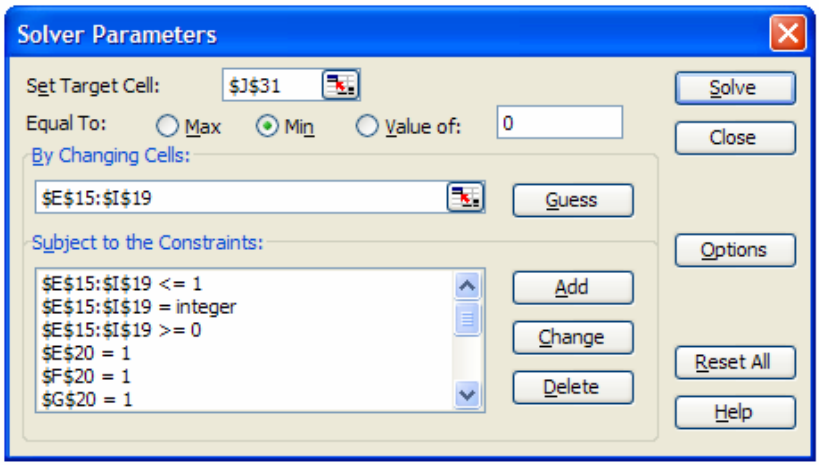
Each load availability constraint is represented in the Solver Parameters box as shown below in the "Subject to the Constraints" section.



Each constraint references the column and cell with the sum of the decision variables for each available load. Notice in the solution that each available load column has four "0" values and only one "1" value, indicating that only one tractor has been assigned to each available load.

	B	C	D	E	F	G	H	I	J
10									
11									
12			**Optimized Solution**	Load Assignments					Total
13				**Load Number**					Truck
14			**Tractor Number**	A	B	C	D	E	Assignments
15			1	0	0	0	1	0	=SUM(E15:I15)
16			2	0	1	0	0	0	=SUM(E16:I16)
17			3	1	0	0	0	0	=SUM(E17:I17)
18			4	0	0	0	0	1	=SUM(E18:I18)
19			5	0	0	1	0	0	=SUM(E19:I19)
20			Load Assignments	=SUM(E15:E19)		=SUM(G15:G19)			=SUM(J15:J19)

The setup of the optimization model in the Solver is now complete. Each constraint in the original formulation is now represented in the spreadsheet Solver design. The optimized solution is generated by clicking the "Solve" button in the top right corner.



The decision variables in the spreadsheet are automatically populated with the optimal solution as shown below. Each load is now optimally matched with an available truck and total empty miles have been minimized.

Final Solver Optimization Output

Current Network	Empty Miles from Tractor Location to Load Pick-up Location				
	Load Number				
Tractor Number	A	B	C	D	E
1	95	150	60	77	123
2	210	160	140	110	190
3	41	15	79	101	40
4	88	91	115	62	55
5	350	210	140	120	105

Optimized Solution	Load Assignments					Total Truck
	Load Number					
Tractor Number	A	B	C	D	E	Assignments
1	0	0	0	1	0	1
2	0	1	0	0	0	1
3	1	0	0	0	0	1
4	0	0	0	0	1	1
5	0	0	1	0	0	1
Load Assignments	1	1	1	1	1	5

Solution Results	Total Empty Miles					Total
	Load Number					Empty
Tractor Number	A	B	C	D	E	Miles
1	-	-	-	77	-	77
2	-	160	-	-	-	160
3	41	-	-	-	-	41
4	-	-	-	-	55	55
5	-	-	140	-	-	140
Miles	41	160	140	77	55	**473**